Five hundred and fifty-seventh Meeting.

November 8, 1865. — STATUTE MEETING.

The PRESIDENT in the chair.

Mr. Safford presented the following paper: —

On the Right-Ascensions observed at Harvard College Observatory in the Years 1862–1865. By T. H. SAFFORD.

IT is part of every astronomer's duty to assure himself in some way of the accuracy of the elements upon which the reduction of his observations depend. If he is a meridian observer, he must make sure that the right-ascensions of the clock- and polar-stars which he employs are correct; and the most thorough means of so doing is to determine them by his own observations.

It is true that this process requires much time and patience; that we, in America, are tempted to think that it has all been so excellently done abroad, that anything we can do will not add to the accuracy of the determinations we derive from foreign sources. I think this notion is somewhat ill-founded. The quantities in question are, as above stated, the right-ascensions of standard stars; they must be predicted, and are so predicted for many years in advance; and the simple formula by which this prediction has to be made,

$$\begin{aligned} a' &= a + \frac{a - a^0}{t - t^0}\,(t' - t) + \Delta p\,\frac{(t' - t^0)\ (t - t^0)}{2} \\ &= a\,\frac{t' - t^0}{t - t^0} - a^0\,\frac{t' - t}{t - t^0} + \Delta p\,\frac{(t' - t^0)\ (t - t^0)}{2}, \end{aligned}$$

shows that not only do the errors of the modern observations come in with more than their full amount, but are somewhat increased by a part of those of the ancient ones.

It ought also to be considered that the observations made with any meridian instrument are liable to errors of obscure origin, which make it somewhat better to use its own results to reduce other observations obtained with it; and that, on the other hand, each good instrument well used, and each well-trained observer, contributes something to the general foundations of the science which no other observer or instrument can do.

The eminent German, English, Russian, and French astronomers have followed this plan within the present century.

Admirable fundamental catalogues have appeared from the observatories of Königsberg, Åbo, Dorpat, Cambridge (Eng.), Greenwich, Edinburgh, and Paris; and two from Pulkova and one from Leiden are to appear within a few years.

I wish to call the attention of American observers to this subject, and will give a short sketch of the process, and enumerate the systematic errors which must be guarded against, and their sources.

First, it is assumed that an approximate catalogue of time-stars is at hand; the first object is to obtain a correct right-ascension of Polaris, peculiar it may be in some degree to observer and instrument, and this is done by opposite culminations, at all seasons; or at two opposite seasons, as spring and autumn. If the values for spring and for autumn differ by any sensible amount, the instrument is ill-mounted, but a constant difference of three seconds of time here did not prevent Bessel from obtaining a value accurate to $0^s.3$. In our Cambridge observations of Polaris, this difference has been much less; in fact hardly sensible.

The next step is to derive the right-ascensions of other polar stars from that of Polaris; and it is here necessary to observe at two opposite seasons of the year, or, which is the same thing, at two opposite culminations.

Now comes the comparison of time stars among themselves, and the best method, and the method which must in all cases form the ultimate test, is to begin by comparing at two opposite seasons the stars Castor, Procyon, and Pollux with the stars γ, α, β Aquilæ; and, finally, to compare the remaining fundamental stars (Maskelyne's 36, for example) with both these groups, though oftener with the nearest one. But this is a method which requires a very perfect clock, especially with perfect compensation, or else not exposed to changes of temperature. In case it is not practicable, we may content ourselves for a time with the more ordinary process of assuming the freedom from constant error of large groups of stars, and thus getting the individual errors of the assumed fundamental catalogue; and the only objection which I know to it is, that being almost universally employed, and Bessel's Fundamenta being the old Catalogue always used for proper motion, we may in this way accumulate dangerous errors, sensibly identical, in nearly all modern observations. It is, therefore, desirable that this point, too, should be tested oftener than it is.

At the Observatory of Harvard College, a fundamental catalogue

of about three hundred and twenty stars is now in process of reduction, which depends on nearly ten thousand observations made since 1862. The instrument has shown a stability of mounting sufficient for an independent and accurate determination of thirty-five polar stars, while it has also proved itself capable of being used for exact right-ascensions of time-stars by the excellence of its pivots, and its general stiffness.

The catalogue will be, if I am not mistaken, the first American Fundamental Catalogue of Right-Ascensions; I can venture to call it so, although the still unfinished normal clock, and the lack of a good circle for solar observations, compel some reliance upon the general accuracy of other determinations. But with the requisite apparatus, about one thousand more observations would free us from even this necessity.

Mr. Oliver presented the following paper: —

On some Focal Properties of Quadrics. By J. E. OLIVER.

I. Any two quadrics have one, and usually but one, common autopolar tetrahedron, **T**. Referred to this their equations become

$$U = a\,w^2 + b\,x^2 + c\,y^2 + d\,z^2 = 0,$$
$$\Upsilon = \alpha\,w^2 + \beta\,x^2 + \gamma\,y^2 + \delta\,z^2 = 0,$$

whether in tangential or in point-coördinates. Using tangential coördinates, all the quadrics

$$U + \lambda\,\Upsilon = (a + \lambda\,\alpha)\,\omega^2 + \ldots\ldots = 0 \qquad (1)$$

have a common enveloping developable; and the entire system is determined by any two of its quadrics [or by any eight of its developable's planes which are not specially related].

The four quadrics that correspond to

$$\lambda = -\frac{a}{\alpha},\ \lambda = -\frac{b}{\beta},\ \lambda = -\frac{c}{\gamma},\ \text{and}\ \lambda = -\frac{d}{\delta},$$

are *plane conics* in the respective planes of reference.

Deforming **T** and (1) together till **T** has one plane at infinity and the other three mutually orthogonal, and then lengthening in suitable proportions the three sets of principal axes, the rectangular point-equation of the system becomes

$$\frac{x^2}{A^2 + k^2} + \frac{y^2}{B^2 + k^2} + \frac{z^2}{C^2 + k^2} = 1, \qquad (2)$$

which is a homofocal system. Three of the plane curves in (1) have become the focal conics in (2). And the fourth curve in (1) becomes in (2) the *spherical circle at infinity*, i. e. the intersection of any finite and finitely-distant sphere, with the plane at infinity, **P**.

For since the cone,

$$x^2 + y^2 + z^2 = 0, \tag{3}$$

which envelops that circle, is sensibly asymptotic to the surfaces (2) when k is indefinitely large, the common tangent-planes to those surfaces then differ not sensibly from the cone's tangent-planes, and hence envelop the circle.

Hence, as Salmon shows, a quadric's three focal curves, with the spherical curve at infinity, are the intersections of non-consecutive rays of that developable which envelops both it and the spherical curve. And they are the *only* intersections; for, being of the fourth degree, the developable cannot cross one of its own rays more than four times.

Any deformation that destroys a sphere, destroys with it the circle at infinity and the homofocalism of system (2), unless it replace that circle by one of (2)'s focal conics; hence the system has but four projective forms. And since, as its tangential equation shows, any four of its quadrics divide each ray of the developable in the anharmonic ratio of their k^2s, this ratio must remain after deformation; hence in the respective forms, k^2 is, at the respective plane curves,

$$\begin{aligned}
= -A^2,\ -B^2,\ -C^2,\ \infty^2,\\
-B^2,\ -A^2,\ \infty^2,\ -C^2,\\
-C^2,\ \infty^2,\ -A^2,\ -B^2,\\
\infty^2,\ -C^2,\ -B^2,\ -A^2;
\end{aligned}$$

hence the four plane conics *exactly replace* one another; and so do their four included groups of quadrics, since projection breaks no cyclic order.

II. The system (2) touches every point of space three times, every line twice, every plane once; except that it meets each ray of the developable *throughout*. A line's two planes of contact with the system are known to be mutually perpendicular, so that the boundaries of two homofocals are seen from any point to intersect at right angles if at all. For if the line touch

$$\frac{x_1^2}{A^2 + k_1^2} + \dots\dots = 1, \qquad \frac{x_2^2}{A^2 + k_2^2} + \dots\dots = 1,$$

at $(x_1 y_1 z_1)$ and $(x_2 y_2 z_2)$ respectively, either point is on the other's polar as to each quadric; or,

$$\frac{x_1 x_2}{A^2 + k_1^2} + \ldots\ldots = 1, \qquad \frac{x_1 x_2}{A^2 + k_2^2} + \ldots\ldots = 1,$$

whose difference,

$$\frac{x_1 x_2}{(A^2 + k_1^2)(A^2 + k_2^2)} + \ldots + \frac{z_1 z_2}{(C^2 + k_1^2)(C^2 + k_2^2)} = 0, \tag{4}$$

is the condition that the two tangent-planes are perpendicular.

Or thus. The pairs of tangent-planes drawn from the line $(x_1 y_1 z_1)$, $(x_2 y_2 z_2)$, to the different surfaces, are known to form an involution, whose double planes, namely the tangents at $(x_1 y_1 z_1)$ and $(x_2 y_2 z_2)$, must form a harmonic pencil with the tangent-planes to any one surface, e. g. to the spherical circle at infinity, and hence are orthotomic.

Particular cases of this orthotomism are that of the three quadrics through one point, and the circularity of the cone mentioned in § IV.

Now this orthotomism would preclude the existence of a common developable; but it fails for the envelope's rays; for since every tangent-plane from a finitely-near point to the circle at infinity has some infinite direction-cosines, while

$$D_{N_1}\ U_1 \cdot D_{N_2}\ U_2 \cdot \cos a_1 \cdot \cos a_2 = \frac{x_1 x_2}{(A^2 + k_1^2)(A^2 + k_2^2)}, \text{ \&c.},$$

are finite, $[U_1, U_2, N_1, N_2, a_1, \ldots \gamma_2$, being the quadrics, their normals, and the direction-angles of these,$]$ — it must be that

$$D_{N_1}\ U_1 \cdot D_{N_2}\ U_2 = 0;$$

hence (4), which is the same as

$$D_{N_1}\ U_1 \cdot D_{N_2}\ U_2 \cdot (\cos a_1 \cos a_2 + \cos \beta_1 \cos \beta_2 + \cos \gamma_1 \cos \gamma_2) = 0,$$

becomes merely identical. And the other demonstration fails, because the two tangent-planes to the circle at infinity coincide.

III. According to Salmon, each ray to a spherical point at infinity is "*perpendicular to itself*" (whatever that may mean); which would extend the above orthotomism to the developable; a result opposite to ours in statement, but probably the same in its actual consequences. Such lines must often thus simulate self-perpendicularity, from their infinite direction-cosines having zero co-factors; and this may make

Salmon's interpretation often convenient and suggestive, though I think it is arbitrary.

If a finitely-near plane [or line] should make an angle θ with itself, it would doubtless touch [or meet] a sphere's circle at infinity; for we should have

$$\cos^2 a + \cos^2 \beta + \cos^2 \gamma = \cos \theta,$$

while

$$\cos^2 a + \cos^2 \beta + \cos^2 \gamma = 1,$$

$$\therefore 0 \cdot \cos^2 a + 0 \cdot \cos^2 \beta + 0 \cdot \cos^2 \gamma = 1 - \cos \theta,$$

$$\therefore \text{some of } (\cos a, \cos \beta, \cos \gamma) = \infty,$$

which is the condition of such contact; [nor need θ nor ∞ be real.] But the converse fails; for

$$\cos a = \infty$$

does not imply

$$\cos^2 a + \cos^2 \beta + \cos^2 \gamma \mp 1.$$

Nor, if θ *can* be > 0, need it be always $90°$. Various considerations often suggest $90°$, as in Salmon's beautiful instance of a circle's tangent-radius. But should not the common tangent of circles

$$\left.\begin{aligned} x^2 + y^2 = 1, \\ x^2 + y^2 = 4, \end{aligned}\right\} \text{ found thus, } \left\{\begin{aligned} x \cos a + y \sin a &= 1, \\ x \cos a + y \sin a &= 2, \\ \therefore \cos a &= \infty, \end{aligned}\right.$$

make in like manner such angles with itself as the circles make with each other's ordinary tangents, namely,

$$60°, \quad \text{and } (\log \overline{2 + \sqrt{3}}) \cdot \sqrt{-1} \quad ?$$

Of course in either instance, to throw the self-contradiction, instead, upon the circle's angle ϵ with its radius vector, we need only regard it as the limiting case of another curve or of an eccentric circle, so that ϵ may be a function of the independent polar angle ϕ.

IV. If quadrics U, V, W, expressed in tangential coördinates, have a common developable envelope, so have $U + \lambda \Upsilon$, $V + \mu \Upsilon$, W; μ being a linear function of λ; for the equation

$$l\, U + m\, V + W = 0,$$

implies

$$l\,(U + \lambda \Upsilon) + m \left(V - \frac{l \lambda}{m} \Upsilon\right) + W = 0.$$

If now Υ be the spherical circle at infinity, and W two separate or coincident points, we see that *when U has double contact with V, or envelopes it, so does every homofocal to U some homofocal to V;* and all the planes of contact intersect in one line or point.

U may have 2, 3, or 4 double contacts with the surfaces $V + \mu \Upsilon$, since the condition

$$l \cdot U + m \cdot V + m \mu \cdot \Upsilon = \text{product}$$

is equivalent to three quadric conditions among $(l, m, m \mu)$, which are satisfied in from 0 to 4 ways, just as three conics have from 0 to 4 common intersections. 1 of these double contacts may be replaced by 1 envelopment; or all 4 by 2 envelopments if (U, V) be cones. *If* U *have* p *double contacts and* q *envelope-contacts with surfaces* $\mathrm{V} + \mu \Upsilon$, *so has* V *with surfaces* $\mathrm{U} + \lambda \Upsilon$.

Of the fact that homofocals envelop homofocals, a familiar case is that each focal line of a cone U touches either co-planar focal of any enveloped quadric $V;$ whence the circularity of that enveloping cone whose vertex is on a focal; and the consequent linear relation among the four distances of two points on one focal from two on the other, &c. Other known cases are, that U's focal lines, when not thus co-planar, are generators of some homofocal to $V;$ and that when U is a quadric of revolution, its foci lie upon V's focals. For in neither of these three cases could the focal curve of U otherwise envelop any homofocal of V.

P. S. Feb. 10, 1866. — The above was in the printer's hands, before I was aware how much of it had been given by Salmon; but I retain it with some changes, as certain points in it may still be new. — J. E. O.

The following gentlemen were elected members of the Academy, viz.: —

J. Victor Poncelet, of Paris, was elected a Foreign Honorary Member, in Class I., Section 4, in place of the late M. Struve.

Mr. Lewis M. Rutherfurd, of New York, was elected Associate Fellow, in Class I., Section 3.

Mr. Samuel Eliot, of Boston, was elected a Resident Fellow, in Class III., Section 3, and Mr. G. W. Hill, of Cambridge, a Resident Fellow, in Class I., Section 2.